PRACTICAL WEEK 2
Analysis of differential equations

Problem 1

Find a value for m such that the function $y = e^{mx}$ is a solution of the given differential equations

$$(1) \qquad y' + 2y = 0,$$
$$(2) \qquad y'' - 5y' + 6y = 0$$

Problem 2

Verify that the expression $x^3y^3 = x^3 + 1$ is an implicit solution of the differential equation: $x\frac{dy}{dx} + y = \frac{1}{y^2}$.

Problem 3

Determine whether the theorem of existence and uniqueness guarantees the differential equation

$$y' = \sqrt{y^2 - 9}$$

possesses a unique solution through the given points

(a) (1,4)

(b) (2,-3)

Problem 4

Verify that

$$y_1(x) = x^2 + c_1 \quad \text{and} \quad y_2(x) = -x^2 + c_2$$

are one-parameter families of solutions to the differential equation

$$(y')^2 = 4x^2$$

Sketch the solution curves.

Problem 5

a) Draw an approximate representation of the direction field of the DE:

$$\frac{dy}{dx} = x(y-4)^2 - 2$$

by calculating it in few representative points in the x-y plane. Describe the slopes of the line elements for $x = 0$, $y = 3$, $y = 4$,and $y = 5$.

b) Graphically solve the initial-value problem $y(0) = 4$. Can a solution $y(x) \to \infty$ as $x(x) \to \infty$? Discuss the solution.

Problem 6

Find the critical points and phase portrait of the two autonomous first-order DE:

$$(1) \qquad \frac{dy}{dx} = 10 - 3y - y^2,$$

$$(2) \qquad \frac{dy}{dx} = y\ln(y+2)$$

Classify each critical point as asymptotically stable, unstable, or semi-stable.

Sketch the solution curve.