MTH2003M Complex Analysis **Practical 4**

1. (a) By using partial fractions and deforming the contour C onto a small circle centred at $z = 3i$, evaluate the integral

$$I = \oint_C \frac{1}{z^2 + 9} \, dz$$

in the case when the point $3i$ lies inside C and the point $-3i$ lies outside it.

(b) Using a similar method, evaluate I in the case when the point $-3i$ lies inside C and the point $3i$ outside it.

(c) Draw a deformed contour that would be suitable for evaluating this integral when the points $\pm 3i$ both lie inside C, and use your results from (a) and (b) to calculate the value of I in this case.

In questions 2 to 5, use Cauchy's integral formula or Cauchy's integral formula for the derivatives as appropriate.

2. Evaluate $\oint_C \frac{e^z}{z-2} \, dz$ if C is

   a) the circle $|z| = 3$;
   b) the circle $|z| = 1$.

3. Evaluate the integral $\oint_C \frac{e^z}{z^2 + a^2} \, dz$, in the case when the contour C contains the points $z = \pm ia$, where a is a real constant. Express your answer in terms of a trigonometric function.

4. Evaluate the integral $\oint_C \frac{ze^z}{(z-a)^3} \, dz$ if the point a (where a is a complex constant) lies within the contour C.

Note: *Try to do question 5 without using partial fractions.*

5. Evaluate the integral $\oint_C \frac{e^z}{z(1-z)^3} \, dz$ in the cases when

   a) the point 0 lies inside the contour C and the point 1 outside it;
   b) the point 1 lies inside C and the point 0 outside it;
   c) the points 0 and 1 both lie within C.