MTH2003M Complex Analysis **Practical 3**

1. Confirm that the function $f(z) = (z-1)^{-1/2}$ has a branch point at $z=1$, and find two possible branch cuts that will make the function single-valued.

2. Show that the integral of $f(z) = z^2 + 1$ around the unit circle (a circle of radius 1) centred on the origin is equal to zero.

3. Integrate $f(z) = z$ along the curve $z = 1 + (i-1)t$, where t is a real parameter such that $0 \le t \le 1$.

4. Evaluate $\int_C \bar{z}\,\mathrm{d}z$ from z = 0 to z = 4 + 2i along the curve C given by

   (a) $z = t^2 + it$;
   (b) the line from z = 0 to z = 2i and then the line from z = 2i to z = 4 + 2i.

5. Show that the inverse sine and tangent functions can be written as

   (a) $\sin^{-1} z = -i \ln\left(iz + \sqrt{1-z^2}\right)$

   (b) $\tan^{-1} z = \dfrac{i}{2}\left[\ln(1-iz) - \ln(1+iz)\right]$

6. By adapting the derivation given in the lectures, show that an alternative formula for the inverse cosine function is $\cos^{-1} z = \dfrac{\pi}{2} + i\ln\left(iz + \sqrt{1-z^2}\right)$.

7. Complete the derivation of Green's theorem started in the lectures by showing that

$$\iint_R \frac{\partial Q}{\partial x}\,\mathrm{d}x\,\mathrm{d}y = \oint_C Q\,\mathrm{d}y,$$

   where $Q(x,y)$ is a real function that, along with all its first partial derivatives, is continuous in the closed set R consisting of a simple closed contour C plus its interior.