MTH2003M Complex Analysis **Practical 2**

1. By attempting to calculate the limit $\lim_{\Delta z\to 0}[f(z+\Delta z)-f(z)]/\Delta z$ with Δz purely real and then with Δz purely imaginary, show that $f(z)=\operatorname{Im} z$ is not differentiable anywhere.

2. Confirm that the function whose real and imaginary parts are given by

$$u(x,y)=\sin x\cosh y \text{ and } v(x,y)=\cos x\sinh y$$

satisfies the Cauchy-Riemann equations for all *z*.

3. Find the harmonic conjugate $v(x,y)$ of each of the following functions, and find the analytic function $f(z)$ of which *u* and *v* are the real and imaginary parts:

(a) $u(x,y)=e^{-x}\cos y$ (b) $u(x,y)=y^2-x^2$

(c) $u(x,y)=\dfrac{y}{x^2+y^2}$

4. Confirm that the functions whose real and imaginary parts are given below satisfy the Cauchy-Riemann equations on the regions specified, and write each function as a function of *z* only.

(a) $u(x,y)=x^3-3xy^2$; $v(x,y)=3x^2y-y^3$ for all $z\in\mathbb{C}$
(b) $u(x,y)=x/(x^2+y^2)$; $v(x,y)=-y/(x^2+y^2)$ for $x^2+y^2\neq 0$

5. Use the Cauchy-Riemann equations to show that the following are not differentiable anywhere:

(a) $f(z)=\operatorname{Im}(z)$ (b) $f(z)=|z|$ (c) $f(z)=\arg(z)$

6. Find the values of *z* for which the following functions satisfy the Cauchy-Riemann equations:

(a) $w=z^2$ (b) $w=|z|^2$

7. A function is defined as $f(z)=\ln r+i\theta$ for $-\pi/2<\theta<\pi/2$.
(a) Convert this function to Cartesian coordinates.
(b) Show that it satisfies the Cauchy-Riemann equations for all *z* such that $\operatorname{Re} z>0$.