MTH2003M Complex Analysis **Practical 1**

1. Write each of the following complex numbers in exponential form:

   (a) -1 (b) $-i$ (c) $1+i$ (d) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ (e) $\frac{1}{2}-\frac{\sqrt{3}}{2}i$

2. Write each of the following complex numbers in the form $a+bi$, where a and b are real:

   (a) $e^{2+i\pi/2}$ (b) $\frac{1}{i}$ (c) $\frac{1}{1+i}$ (d) $(1+i)^3$ (e) $|3+4i|$

3. Sketch the following regions in the complex plane, using full lines for boundaries that are included in the region and dashed lines for boundaries that are excluded:

   (a) $|z| \leq 1$ (b) $|2z+1+i| < 4$ (c) $\operatorname{Re} z \geq 4$

   (d) $|z| \leq |z+1|$ (e) $0 < |2z-1| \leq 2$ (f) $\pi/4 < \arg z \leq 3\pi/4$

4. Show that, for two complex numbers *z* and *w*, $\overline{z+w} = \bar{z}+\bar{w}$ and $\overline{zw} = \bar{z}\,\bar{w}$.

5. Show that, for any complex number *z*,

   $$\operatorname{Re}(z) = \frac{1}{2}(z+\bar{z}) \qquad \text{and} \qquad \operatorname{Im}(z) = \frac{1}{2i}(z-\bar{z}).$$

   Use your result to express the equation of the line $x+3y=2$ in terms of z and $\bar{z}$.

6. Find the three cube roots of *i*, writing your answers in exponential, polar and standard form.

7. Solve for the roots of the following equations, writing your answers in exponential form:

   (a) $z^3 = 4$ (b) $z^4 = -1$ (c) $z^4 + 2z^2 + 2 = 0$

8. Solve the equation $(az+b)^3 = c$, where *a*, *b* and *c* are real, positive constants.

	0	**π/6**	**π/4**	**π/3**	**π/2**
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	*