MTH2004M Differential Equations

Chapter 2 First Order Differential Equations

- Geometrical Interpretation of Solutions
- Separable Differential Equations
- Linear Differential Equations

Recall: Check List for First Order Equations

Tackling differential equations check list:

1. **Classify** our differential equation
   - *first-order, ordinary:* $\frac{dy}{dx} = f(x, y)$
2. Choose a **method** of solution
   - *separation of variables:* $\frac{dy}{dx} = g(x)h(y)$
3. Derive **implicit** of **explicit** families of solutions
4. Apply an **initial condition** to get a particular solution
5. Sketch/plot the **solution curve**
6. Determine the **interval of validity**

Separable Differential Equations

Classification

- Ordinary - derivatives with respect to one variable
- First order - highest derivative is order one

Definition A first order ordinary **separable** differential equation has the form

$$\frac{dy}{dx} = g(x)h(y)$$

Separation of Variables

<u>Definition</u> A **separable** differential equation has the form

$$\frac{dy}{dx} = g(x)h(y)$$

We like separable equations, because we can rearrange them and simply **integrate** to solve

$$\begin{aligned} \frac{1}{h(y)}dy &= g(x)dx, \text{ first rearrange} \\ \Rightarrow \int \frac{1}{h(y)}dy &= \int g(x)dx, \text{ then integrate} \\ \Rightarrow H(y) &= G(x) + c \end{aligned}$$

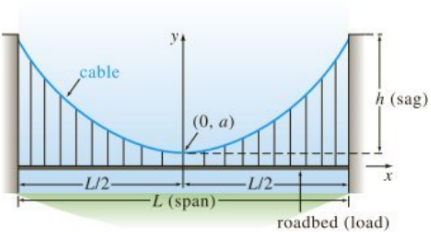
Here $H(y)$ is the anti derivative of $1/h(y)$
and $G(x)$ is the anti derivative of $g(x)$

Suspension Bridges Consider the IVP

$$\frac{dy}{dx} = \frac{\rho x}{T}; \qquad y(0) = a$$

This is model for the shape of a suspension bridge, where ρ is the weight of the cable and T is the tension to be determined.

Solve the differential equation to determine the shape of the bridge.



What other condition can we apply to find T?

Modelling with Separable Equations

Chemical Reactions Consider the IVP

$$\frac{dA}{dt} = -A^2; \qquad A(0) = A_0$$

This is a model for some substance A reacting into some substance B.

We require $A(t) + B(t) = A_0$ to conserve mass.

Solve the differential equation to find the time at which 50% of the substance has reacted.

Check your answer by plotting $A(t)$ and $B(t)$ for $A_0 = 1$ on desmos.

Population Model Consider the IVP

$$\frac{dP}{dt} = k \cos t \qquad P(0) = 1$$

where k is a positive constant.

This is model for some population $P(t)$.

By solving the differential equation, consider what kind of population this describes?

Use desmos to understand how k influences the population dynamics.

Modeling with Separable Equations

Cooling a Cake Newton's law of cooling is given by

$$\frac{dT}{dt} = k(T - T_a),$$

where k is thermal diffusivity and T_a is the ambient temperature.

A cake is removed from the oven at $150^o C$. Three minutes later the temperature is $90^o C$.

Solve the ODE to calculate thermal diffusivity and consider how long it will take a cake to cool to room temperature.

Next week

Tackling differential equations check list:

1. **Classify** our differential equation
   - *first-order, ordinary:* $\frac{dy}{dx} = f(x, y)$
2. Choose a **method** of solution
   - *separation of variables:*
   - *integration factor method*
3. Derive **implicit** of **explicit** families of solutions
4. Apply an **initial condition** to get a particular solution
5. Sketch/plot the **solution curve**
6. Determine the **interval of validity**