Practicals, Week 2

2nd academic week, 4 Oct 2024
Distance Theorem, Parameters, Chance of correct decoding

Exercises to be discussed in the practicals session

Problem P1 (Distance Theorem and Nearest Neighbour Decoding)

Consider the following 3-ary code:

$$C = \{00000,\ 11222,\ 22111\} \subset \mathbb{F}_3^5.$$

a) Explain why C detects up to four errors.

b) Explain why C corrects up to two errors.

c) Find a word $w \in \mathbb{F}_3^5$ that has Hamming distance exactly three to two different codewords in C.

d) Conclude from item (c) that C does not always correct three errors.

Problem P2 (Prescribing error correction and error detection)

Suppose you are starting to design a code that has to detect or correct specific numbers of errors.

a) How small can the minimal distance of this code be if you need it to detect up to 5 errors?

b) How small can the minimal distance of this code be if you need it to correct up to 3 errors?

Problem P3 (Parameters)

Determine the parameters n, M, d and q of the code $C = \{00000,\ 11222,\ 22111\}$ from Exercise P1.

Problem P4 (Repetition codes and parameters)

Consider the 5-ary alphabet $A = \mathbb{F}_5 = \{0, 1, 2, 3, 4\}$ and the space of all messages $\mathbb{F}_5^2$ of length two over A.

a) Write down the repetition code C obtained by repeating **twice** the symbols of all messages in $\mathbb{F}_5^2$.

b) Determine the parameters n, M, d and q of the code C from item (a).

Problem P5 (Hamming's original code)

The following table lists all codewords of the original code proposed by R. Hamming:

0000000	1000110
0001111	1001001
0010011	1010101
0011100	1011010
0100101	1100011
0101010	1101100
0110110	1110000
0111001	1111111

a) Determine the four parameters n, M, d and q of Hamming's code.

b) Prove that Hamming's code detects two errors and corrects one error.

Problem P6 (Chance of correctly decoding)

Let the code $C = \{000, 111\}$ be given, used in a symmetric channel with symbol error probability p.

a) Determine its parameters.

b) Suppose a codeword $c \in C$ is sent and a word w is received. Compute the probability of **correctly** decoding w.

c) How much is, in percentage, the probability of decoding w **incorrectly** if $p = 5\%$? And how much is it if $p = 1\%$?